

## 02 Fractions and Decimal Numbers

### MAJORITY AND MATHEMATICS

Majority is the name used to say the proportion of necessary votes to take decisions in a vote.

But there are different types of majority according to the importance of the decision.

#### ***Simple majority***

It is elected the most voted option. The number of votes cast and abstentions don't matter.

#### ***Absolute majority***

It is necessary to get more than a half of all members.

#### ***Qualified majority or supermajority***

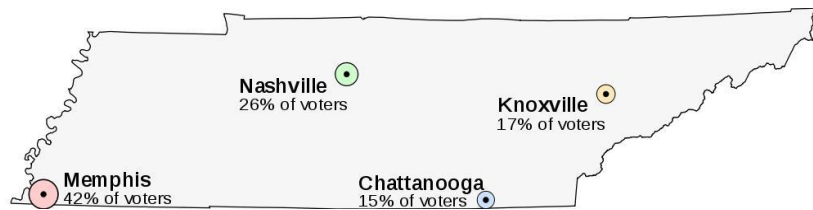
It is necessary an established percentage of votes. For instance, it could be two thirds, three fifths or four sevenths.

This method is used in important matters.

#### ***Capital of Tennessee***

Imagine that Tennessee is having an election on the location of its capital.

The population of Tennessee is concentrated around its four major cities, which are spread throughout the State.



For this example, suppose that the entire electorate lives in these four cities and that everyone wants to live as near to the capital as possible.

The candidates for the capital are:

- Memphis, the state's largest city, with 42% of the voters, but located far from the other cities.
- Nashville, with 26% of the voters, near the center of the State.
- Knoxville, with 17% of the voters.
- Chattanooga, with 15% of the voters.

The preferences of the voters would be divided like this:

42% of voters (close to Memphis)	26% of voters (close to Nashville)	15% of voters (close to Chattanooga)	17% of voters (close to Knoxville)
1. <b>Memphis</b> 2. Nashville 3. Chattanooga 4. Knoxville	1. <b>Nashville</b> 2. Chattanooga 3. Knoxville 4. Memphis	1. <b>Chattanooga</b> 2. Knoxville 3. Nashville 4. Memphis	1. <b>Knoxville</b> 2. Chattanooga 3. Nashville 4. Memphis

Here, Memphis has a simple majority (42%) of the first preferences, but not an absolute majority.

We can contrast this by looking at the fourth preferences, where the absolute majority of voters (58%) have placed Memphis last.

What do you propose to solve the problem in the best way?

## 1. FRACTION OPERATIONS

### ADDITION AND SUBTRACTION

Adding and subtracting fractions is a process of collecting together fractions with the same denominator. Then just add or subtract the numerators.

$$\frac{7}{3} + \frac{-5}{3} =$$

When the denominators are different we have to form equivalent fractions to make the denominators the same. This process is the reduction to common denominator.

$$\frac{7}{5} + \frac{-2}{3} =$$

The best way to reduce to common denominator is to find out the lcm of the denominators.

$$-\frac{2}{9} - \frac{1}{6} + \frac{3}{4} = \text{---} - \text{---} + \text{---} =$$
$$lcm(9; 6; 4) =$$

### *Opposite fraction*

It is the fraction we get by changing the sign:

$$\text{Opposite of } \frac{7}{3} =$$

$$\text{Opposite of } \frac{-2}{3} =$$

### MULTIPLYING AND DIVIDING FRACTIONS

The product of two fractions is the fraction that has as numerator the product of two numerators and as denominator the product of two denominators.

$$\frac{6}{7} \times \frac{-3}{10} = \text{---}$$

The division of two fractions is the fraction that has as numerator the product of the first numerator by the second denominator and as denominator the product of the first denominator by the second numerator.

$$8 : \frac{10}{13} = \text{---} =$$

### *Inverse fraction*

It is the fraction we get by swapping the numerator for the denominator

$$\text{Inverse of } \frac{7}{3} =$$

$$\text{Inverse of } \frac{-2}{3} =$$

## 2. DECIMAL OPERATIONS

### ADDITION AND SUBTRACTION

We have to operate according to the comma or point position.

For example,

Do  $26.79 + 0.4085 + 4.0 + 528.254$

Take away 0.508 from 83.5

Handwritten addition of four decimal numbers:

$$\begin{array}{r} 26.79 \\ + 0.4085 \\ + 4.0 \\ + 528.254 \\ \hline \end{array}$$

Handwritten subtraction of 0.508 from 83.5:

$$\begin{array}{r} 83.500 \\ - 0.508 \\ \hline \end{array}$$

### MULTIPLICATION

We have to multiply in a normal way and then put as many decimal places as the sum of the two numbers have. Complete the multiplication.

Handwritten multiplication of 4.56 by 1.7:

$$\begin{array}{r} 4.56 \\ \times 1.7 \\ \hline 3192 \end{array}$$

### DIVISION

We have to multiply the dividend and the divisor by a power of ten that gets a divisor without decimal part. Then we divide in a normal way.

To get the decimal part for the quotient we have to add a comma or point when the remainder is lower than the divisor.

$$21,66 : 3,8 \longrightarrow$$

## FRACTIONS AND DECIMAL NUMBERS

A fraction has a decimal form. We get it by dividing the numerator into the denominator. For example,  $\frac{8}{5} = 1'6$

In this way we can obtain three types of decimal expression:

An integer if the remainder is zero.

For example,  $\frac{15}{3} = 5$

A finite decimal part if the number of decimal is limited.

For example,  $\frac{7}{4} =$

A recurring decimal is a decimal number that has digits that repeat forever.

For example,  $\frac{13}{11} =$

## APPROXIMATIONS AND ESTIMATIONS

Sometimes we have to leave out some decimal because it has no sense to put a lot of decimal in a number. This value is an approximation. It could be lower than the real value (defect) or higher than the real value (excess).

We have three forms to do that:

1. Truncate a decimal number to a given order is to leave the digits prior to that order and eliminate the others.

Example

Truncate to hundredths 3,14159265... →

Truncate to ten-thousandths 3,14159265... →

2. Approximate the number by defect or excess according to the value we take: lower or greater than the real value.

Example

Approximate to hundredths 3,14159265... by excess →

Approximate to ten-thousandths by defect 3,14159265... →

3. Round a decimal number to a given order is to suit the last digit as the next rule: If the next digit is greater than or equal to 5 increases by one the previous digit; otherwise, it remains equal.

Example:

Round to hundredths 3,14159265... →

Round to ten-thousandths 3,14159265... →

### 3. GENERATRIX FRACTION

#### RATIONAL NUMBERS

They are the numbers we get from the ratio of two integers. We represent them by the Q letter.

#### GENERATRIX FRACTION

It is the fraction corresponding to a decimal number. How to get it?

1. If the decimal part is finite we have to divide all the digits without decimal point into a power of ten.

Example:  $3,25 = \frac{\quad}{\quad}$

2. If the decimal number has a period, we get thus the fraction:
  - The numerator is the result of subtract all the digits (without decimal point) minus the not recurring part.
  - The denominator is formed by writing as many 9 as digits has the period and as many 0 as decimal digits outside of the period it has.

Example:

$3,\overline{7} = \frac{\quad}{\quad}$

$10,\overline{603} = \frac{\quad}{\quad}$

#### IRRATIONAL NUMBERS

They are the numbers we can't obtain by a fraction. That is the numbers with an infinite decimal part without period.

$\pi$  is a notable example of irrational number. It is impossible to write this quantity as a fraction and, therefore, to find a period in its decimal part.

You know  $\pi$  is the perimeter of a circumference whose diameter is 1.

$\pi = 3,1415926535897932384626433832795\dots$

## EXERCISES AND PROBLEMS

### 1. Fraction operations

1. What number is the highest  $\frac{5}{6}$  or  $\frac{6}{8}$ ? Explain your answer.
2. Complete the blank:  $\frac{42}{24} = \frac{\quad}{20}$
3. Draw in the number line the following fractions:  $\frac{7}{3}$  and  $-\frac{11}{3}$
4. I'm going to receive  $\frac{3}{5}$  of an inheritance of 12000 €. How much am I going to have?
5. In a class of 40 people 80% of students have passed an exam. How many students passed the exam?
6. Work out:
  - a) A half of  $\frac{7}{8}$
  - b) One third of  $\frac{9}{5}$
  - c) The triple of a half of  $\frac{2}{3}$
7. Work out mentally:
  - a) Two fifths of 400
  - b) Three sevenths of 140
  - c) The number whose five sixths are 25.
  - d) The number whose two fifths are 160.
8. What fraction of an hour do they represent?
  - a) 15 minutes; b) 20 minutes; c) 10 minutes; d) 12 minutes; e) 120 seconds; f) 45 seconds.

9. Calculate: a)  $2 - \frac{5}{6} + \frac{1}{4}$  b)  $\frac{-1 + \frac{3}{4} - \frac{1}{3}}{2 - \frac{1}{4}} =$

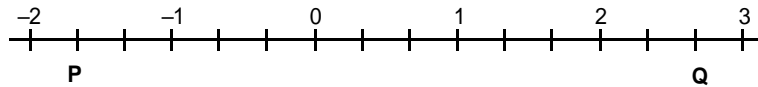
10. Calculate step by step: a)  $\frac{7}{5} : \frac{7}{10} - \frac{5}{6} : \frac{1}{4}$  b)  $\frac{\frac{3}{2} + \frac{1}{4}}{\frac{5}{6} - \frac{1}{3}} =$

11. Work out the following and simplify the resulting fraction:  $\frac{\frac{3}{4} - 2}{\frac{1}{5}} \cdot 3$

12. Work out the following and simplify the resulting fraction:  $\frac{\frac{-3}{5} \cdot 7 + 4 - \frac{1}{2}}{2 - 3 \cdot \frac{3}{5}} =$

13. Work out the following and simplify the resulting fraction:  $\frac{2 - \frac{1}{3}}{1 + \frac{1}{4}} + \frac{\frac{-1}{5} \cdot 3}{\frac{1}{2} + 4} =$

- 14.** What numbers represent the dots P and Q that you see in the following number line:

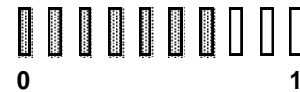


- 15.** Work out the following and simplify the resulting value:
- $$\frac{-5}{3} \cdot \frac{2}{6} + 8$$
- $$\frac{5}{3} - 4 \cdot 3$$

- 16.** Do the following:
- $$\frac{\frac{2}{3} \left( \frac{5}{6} - \frac{1}{3} \cdot \frac{1}{2} \right) + \frac{2}{2} : \frac{3}{5}}{\left[ \frac{1}{2} + \left( \frac{2}{3} : \frac{1}{2} \right) - \frac{5}{6} \right]}$$

- 17.** The fuel tank of a car reads as follows

What fraction indicates the full part? What fraction indicates the empty part?



- 18.** A box contains 60 chocolates. Eva ate  $\frac{1}{5}$  of the chocolates and Anna  $\frac{1}{2}$ .
- a) How many chocolates did Eva and Anna eat?
- b) What fraction of chocolates did they eat together?

- 19.** Write three rational numbers which are between  $\frac{1}{5}$  and  $\frac{3}{10}$

- 20.** How many bottles of  $\frac{3}{4}$  litres does a man need to pack 600 litres of wine?

- 21.** A wire is 72 m long and it is cut into two pieces. One has the five sixths of cable. How many meters does each piece measure?

- 22.** To prepare a cake, you need:
- $\frac{1}{3}$  of a pack of 750 grams of sugar.
- $\frac{3}{4}$  of a kg pack of flour.
- $\frac{3}{5}$  of a block of butter of 200 g.

Find, in grams, the quantities that are needed to prepare the cake.

## 2. Decimal operations

- 23.** Calculate the decimal value of these fractions and say what kind of fractions they are.

a)  $\frac{7}{4}$     b)  $\frac{10}{6}$     c)  $\frac{195}{15}$     d)  $\frac{13}{3}$     e)  $\frac{3}{7}$     f)  $\frac{2}{15}$

- 24.** Classify the fractions as their decimal form: a)  $\frac{7}{4}$     b)  $\frac{10}{6}$     c)  $\frac{13}{3}$     d)  $\frac{3}{7}$

- 25.** Classify the fractions as their decimal form: a)  $\frac{7}{8}$     b)  $\frac{5}{6}$     c)  $\frac{105}{3}$     d)  $\frac{14}{11}$

- 26.** Change:

- a) 200 m/s into km/h
- b) 40 m/s into km/h
- c) 120 km/h into m/s
- d) 65 km/h into m/s

### **3. Generatric fractions. Irrational numbers**

- 27.** Convert the following numbers to a fraction and simplify the result:  
a)  $0,02\bar{5}$  b)  $1'\bar{8}$  c)  $10'\bar{603}$ . Check the answer with the calculator.
- 28.** Convert the following numbers to a fraction:  $0.005\bar{1}$ ;  $0.\bar{051}$ ;  $0.05\bar{1}$ . Check the answer with the calculator.
- 29.** Convert the following numbers to a fraction and simplify the result:  
a)  $1,2\bar{5}$  b)  $8'\bar{3}$  c)  $2'\bar{681}$ . Check the answer with the calculator.
- 30.** The diameter of a wheel of a car is 50 cm. Estimate how many meters the car runs after 100 turns. Write the default value and the excess value.